

# Higher Dimensional Cosmological Model in Space-Time-Mass (STM) Theory of Gravitation

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## Abstract

A new class of non-static higher dimensional vacuum solutions in space-time -mass (STM) theory of gravity is found. This solution represent expanding universe without big bang singularity and the higher dimension of these models shrinks as they expands.

## 1 Introduction

The domination of Newtonian theory had received the blow at the advent of Einstein general theory of relativity, where the Newtonian gravitational parameter  $G$  restricted as constant. Several theories of gravitation ( Brans and Dicke [1], Dirac[2], Hoyle Narlikar[3] and Canuto et al.[4] ) alternatives to Einstein general theory of relativity have been proposed on which the parameter  $G$  and or mass  $m$  vary slowly with time. These theories have the common feature that they assume the absences of fixed scales in cosmological solutions over times comparable to the age of the universe.

In this contents, the variable gravitational theory proposed by Wesson [5, 6] deserved serious attention. The space-time five dimension with coordinate  $x^0 = ct$  ( $c$ =velocity of light,  $t$  = time),  $x^{1,2,3}$  (space coordinate) and  $x^4 = \frac{Gm}{c^2}(m = mass)$ . The Einstein theory is recovered when the velocity  $\frac{Gm}{c^2} \frac{dm}{dt} = 0$ . In some sense, the usual Einstein theory Einstein theory would be embedded in it. This new five dimensional theory of variable rest mass is a natural extension a of relativity theory. In our work, we have extend the work of L K Chi [7] for  $n$ -dimensional variable mass theory of gravity. Here we intend to investigate the new class of vacuum solutions in  $n$ -dimensional space-time-mass (STM) theory of gravity. These

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cosmological higher dimensional models describe expanding universe without a big bang singularity and the higher dimensional of these models shrinks as they expand.

## 1.1 Field Equations

The line element for a n-dimensional homogeneous and spatially isotropic cosmological model is taken as

$$ds^2 = e^\nu dt^2 - e^\omega \sum_{i=1}^{(n-2)} dx_i^2 + e^\mu dm^2 \quad (1)$$

where  $\mu, \omega$  and  $\nu$  are the functions of time and mass. Here the coordinate  $x^0 = t$ ,  $x^{1,2,\dots,(n-2)}$  (space coordinate) and  $x^{(n-1)} = m$ . For simplicity we have set the magnitudes of both  $c$  and  $G$  to unity. By applying this metric to the Einstein field equation  $G_{ij} = R_{ij} - \frac{1}{2}g_{ij} R = 0$ , we get

$$G_{00} = -(n-2)(n-3) \frac{\dot{\omega}^2}{8} - (n-2) \frac{\dot{\omega}\dot{\mu}}{4} - (n-2) e^{\nu-\mu} \left( \frac{\ddot{\omega}^*}{2} - \frac{\dot{\omega}^* \dot{\mu}^*}{4} + (n-1) \frac{\dot{\omega}^{*2}}{8} \right) \quad (2)$$

$$G_{11} = e^{\omega-\mu} \left( \frac{\ddot{\mu}}{2} + \frac{\dot{\mu}^2}{4} - \frac{\dot{\mu}\dot{\nu}}{4} \right) + (n-3) e^{\omega-\nu} \left( \frac{\ddot{\omega}}{2} + \frac{\dot{\omega}\dot{\mu}}{4} - \frac{\dot{\nu}\dot{\omega}}{4} + (n-2) \frac{\ddot{\omega}^2}{8} \right) \\ + (n-3) e^{\omega-\mu} \left( \frac{\ddot{\omega}^*}{2} + \frac{\dot{\omega}^* \dot{\mu}^*}{4} - \frac{\dot{\nu}^* \dot{\omega}^*}{4} + (n-2) \frac{\dot{\omega}^{*2}}{8} \right) + e^{\omega-\mu} \left( \frac{\ddot{\nu}^*}{2} + \frac{\dot{\nu}^{*2}}{4} - \frac{\dot{\nu}^* \dot{\mu}^*}{4} \right) \quad (3)$$

$$G_{11} = G_{22} = G_{33} = \dots = G_{(n-2)(n-2)}$$

$$G_{0(n-1)} = (n-2) \left( \frac{\dot{\omega}^*}{2} + \frac{\dot{\omega}^* \dot{\omega}}{4} - \frac{\dot{\nu}^* \dot{\omega}}{4} - \frac{\dot{\mu} \dot{\omega}^*}{4} \right) \quad (4)$$

$$G_{(n-1)(n-1)} = -(n-2)(n-3) \frac{\dot{\omega}^{*2}}{8} - (n-2) \frac{\dot{\omega}^* \dot{\nu}^*}{4} - (n-2) e^{\mu-\nu} \left( \frac{\ddot{\omega}}{2} - \frac{\dot{\omega}\dot{\nu}}{4} + (n-1) \frac{\dot{\omega}^2}{8} \right) \quad (5)$$

where a dot and star denote, respectively partial derivative with respect to time and mass.

## 1.2 Solutions

Equation (2), (3) and (5) have the peculiar property that the time derivative and mass derivative are completely separated.

Hence we get

$$(n-3) \frac{\dot{\omega}^2}{2} + \dot{\omega}\dot{\mu} = 0 \quad (6)$$

$$(n-3) \frac{\dot{\omega}^2}{2} + \dot{\omega}^* \dot{\nu} = 0 \quad (7)$$

$$\ddot{\omega} - \frac{\dot{\omega}^* \dot{\mu}}{2} + (n-1) \frac{\dot{\omega}^2}{4} = 0 \quad (8)$$

$$\ddot{\omega} - \frac{\dot{\omega} \dot{\nu}}{2} + (n-1) \frac{\dot{\omega}^2}{4} = 0 \quad (9)$$

assuming that  $\dot{\omega}^*$  and  $\dot{\omega}$  are not zero, it then follows from equations (6) and (7)

$$\dot{\omega} = -\frac{2 \dot{\mu}}{(n-3)} \quad (10)$$

$$\dot{\omega}^* = -\frac{2 \dot{\nu}}{(n-3)} \quad (11)$$

with the aid of equations (8) - (11), we find that equation (3) is identically satisfied.

Using equations (10) and (11), we can integrate equation (4) to give

$$e^{\omega} = \left[ \frac{(n-2)}{2} \int k(m) dm + r(t) \right]^{\frac{2}{(n-2)}} \quad (12)$$

where  $k(m)$  is an arbitrary function of  $m$  and  $r(t)$  is an arbitrary function of  $t$ . once  $\omega$  is known,  $\mu$  and  $\nu$  can easily be found from equations (10) and (11).

$$e^{\mu} = \frac{P k(m)^2}{\left[ \frac{(n-2)}{2} \int k(m) dm + r(t) \right]^{\frac{n-3}{(n-2)}}} \quad (13)$$

$$e^{\nu} = \frac{Q r(t)^2}{\left[ \frac{(n-2)}{2} \int k(m) dm + r(t) \right]^{\frac{2}{(n-2)}}} \quad (14)$$

where  $P$  and  $Q$  are integrating constant.

### 1.3 Conclusion

The implication of the condition  $\dot{\omega} = 0$  is that the function  $r(t)$  can not be a constant and these cosmological models are not static. Equation (10) implies that the higher dimensions shrinks as the universe expand. This conclusion can also be drawn from the explicit expansion (12) and (13). It is interesting to note that if  $r(t)$  is an increasing function of time  $t$  and the function  $k(m)$  is bounded, then  $e^{\mu}$  becomes smaller as  $e^{\omega}$  increase. For the expanding universe, we may choose the function  $r(t)$  to be positive function of  $t$  or an exponential function of  $t$ . The condition  $\dot{\omega}^* \neq 0$  implies that we can not choose the arbitrary function  $k(m)$  to be zero. Hence, this expanding universe in higher dimensional STM theory does not start with a big bang singularity.

## References

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